SPUR GEARS

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SPUR GEARS

DESIGN, OPERATION, AND PRODUCTION

BY
EARLE BUCKINGHAM
Associate Professor of Engineering Standards and Measurement,
Massachusetts Institute of Technology

First Edition
Seventh Impression

McGRAW-HILL BOOK COMPANY, Inc.
NEW YORK AND LONDON
1928

7864

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PRINTED IN THE UNITED STATES OF AMERICA

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THE MAPLE PRESS COMPANY, YORK, PA.

PREFACE

The subject of gearing is an old one, yet our knowledge of it is still far from complete. New and more severe requirements that gearing must meet are developing continually. New and improved materials and methods for the production of gears are also being made available. To a great extent, however, the design of gear-tooth profiles has become conventionalized, and many of the fundamentals of gear-tooth action have been lost sight of in this process. As a result, the full benefits of the improved materials and methods of production have not always been secured.

The introduction of the formed cutter for the production of gear teeth imposed upon the manufacturers of these small tools the task of gear-tooth design. As a matter of fact, this was a problem that the majority of other manufacturers was glad to shift to someone else. Certain conventions based upon the limitations of such formed tools were thus introduced into gear-tooth design. Many of such conventions were naturally carried over into the design of gear-tooth forms produced by molding or generating processes, although the limitations of the molding processes are quite different, in many respects, from those of form cutting. For example, as a matter of economy, which is always a controlling one in all industrial processes, the number of different formed cutters required to produce mating gears of all different tooth numbers should be reduced to a minimum. This has required the use of fixed tooth proportions for all gears of any one series or system. With the molding processes, however, and the use of the full-involute form for the gear-tooth profiles, a single cutter will produce mating gears without the limitation of fixed tooth proportions. In general, economy of manufacture of gears requires the standardization and simplification of the tools used to produce the gear teeth rather than the standardization of the gears themselves.

The purpose of this book is to bring out as clearly and simply as possible the fundamental characteristics of spur gears, in the hope that more effective use may be made of the facilities now available for producing them. The attempt has been made to give a complete mathematical exposition of this subject as simply as possible and at the same time to include in the text sufficient explanation, so that a grasp of the subject may also be gained without following through all of the mathematical proofs. To this end, many tables have been included to simplify the use of the material.

The author can claim but little originality on his part for the material published here. His major task has been the selection and the arrangement of the work of many others into such a reference book on this subject as he would desire for himself. In order to make it more nearly complete, certain assumptions are used where exact knowledge is lacking. Where assumptions are made, however, they are plainly stated as such. The author has tried to give due credit in all cases where the original source of the information he has used was known; there are undoubtedly many other cases where this, because of lack of definite information, has not been done.

This book is primarily one on the subject of involute spur-gear teeth. To the author's mind, this subject is naturally divided into three main branches: first, their design; second, their operation; and third, their production. The book is therefore divided into three such sections.

EARLE BUCKINGHAM

CAMBRIDGE, MASSACHUSETTS
January, 1928

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SECTION 1

THE DESIGN OF GEAR-TOOTH FORMS

SPUR GEARS

CHAPTER I

CONJUGATE GEAR TOOTH ACTION

There is an almost infinite number of forms which can be used as gear-tooth profiles. The essential purpose of gear-tooth profiles is to transmit rotary motion from one shaft to another. Usually the additional requirement of uniform motion also exists.

Although the involute profile is the one most commonly used today for gear-tooth forms, occasions arise when some other form of profile must be employed. In addition, there are also other problems than the transmission of rotary motion where a thorough knowledge of the theory of gearing assists in the most direct solution. One of such problems is the hobbing of spline shafts.

Again, in order to appreciate fully the great simplicity of the involute form, both in theory and in its production, it is necessary to have a clear understanding of the principles of conjugate gear-tooth action. We will therefore consider at this time the characteristics of tooth profiles that will transmit through each other uniform rotary motion. The action between such profiles is called *conjugate gear-tooth action*.

In essence, a pair of mating gear-tooth profiles are cams, the one acting against the other to produce the desired relative motion. With certain restrictions, one profile can be chosen at random, and a proper mating profile can be developed.

As a definite example, we will now consider the action between two lever arms, the one with a pin which engages in a slot of the other, the driven member. This is illustrated in Fig. 1.

The lever with the pin is rotating with uniform motion in the direction shown by the arrow. The rate of rotary motion of the driven or slotted lever depends upon the relative lengths of a and b, as shown in the illustration. The driven lever will move b/a times as fast as the driver. If these lengths a and b are equal and remain equal in all contact positions, the rates of rotation will also be equal.

The lengths of *a* and *b* are determined by the intersection of the normal to the mating profiles at the point of contact with the common center line of the axes of the two levers. A *normal* is a line which is perpendicular to the tangent of a curve at its point of tangency. Thus, the normal to a straight line is the perpendicular to it, and the normal to a circle is a radial line.

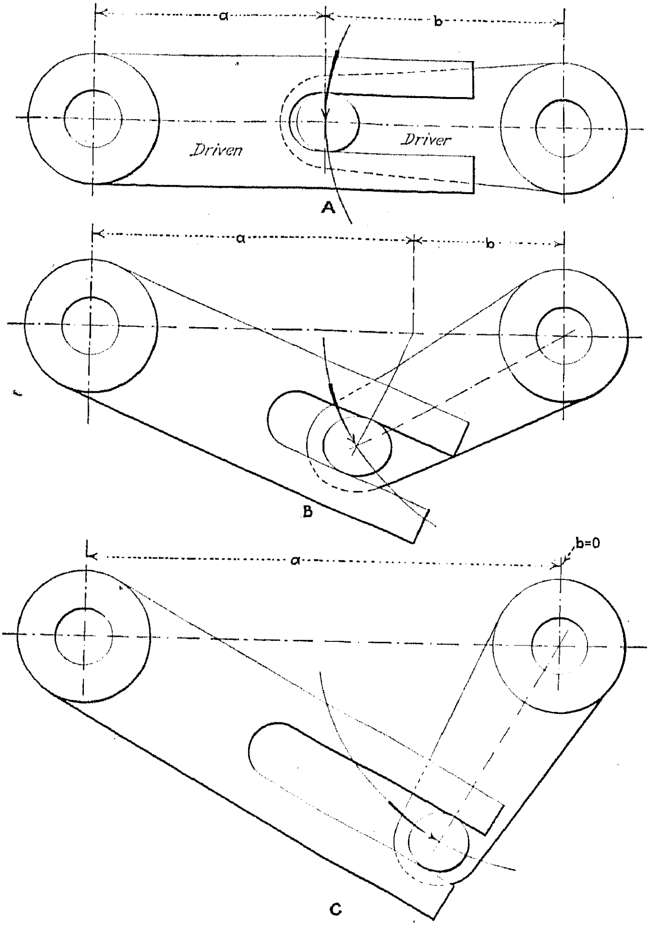

Fig. 1.—Transmission of rotary action through lever arms.

At the top of Fig. 1, position *A*, the distances *a* and *b* are shown as equal. At this position, therefore, the rates of rotation of the two levers are equal. At position *B*, when the driver has moved 30 deg. from its original position, *b* is shorter and *a* is longer than originally; therefore, the driven or slotted lever is moving more slowly than the driver. At position *C*, when the driver has moved 60 deg. from its original position, the length of *b* has become zero, while the length of *a* has become double its original length. At this position, the driven lever has ceased to move. Further motion of the driver, assuming a sufficient length of slot in the driven lever, would cause the driven lever to start to move in the opposite direction.

It is evident that the action between the two levers, shown in Fig. 1, does not result in the transmission of uniform rotary motion from one shaft to the other. In order to transmit this uniform motion, the relative lengths of *a* and *b* must remain constant at all operating positions of the mating profiles. This gives us the basic law of conjugate gear-tooth profiles, which may be expressed as follows:

To transmit uniform rotary motion from one shaft to another by means of gear teeth, the normals to the profiles of these teeth at all points of contact must pass through a fixed point in the common center line of the two shafts.

This fixed point in the common center line is called the *pitch point.* With every gear-tooth form, except the involute, there is a definite pitch line or circle from which the conjugate tooth profiles must be developed. The pitch circles of mating gears must be tangent to each other. The point of tangency of these pitch circles is the pitch point. These pitch circles are of such size that if they were to drive each other by friction, they would transmit the required relative motion. The sizes of these pitch circles are inversely proportional to the rate of rotation; for equal speed, these sizes are equal; for double speed, the pitch circle of the slower gear is twice the size of the faster, etc. The tooth profile may be symmetrical or unsymmetrical in respect to the pitch line; it may be all above it or all below it or partly above and partly below.

As stated before, the profile of one gear may be chosen arbitrarily, and the conjugate profile for the mating gear can be developed. For every conjugate gear-tooth profile there is also a basic-rack form. As mating tooth profiles act together, the point of contact between them will travel along a line or path called the *line of action* or the *path of contact.* Once a pitch line has been established for any profile, a definite line of action exists, along which contact with all other conjugate profiles is made. There is a definite relation between a gear-tooth profile and its line of action, so that if either one is given, the other is fixed. When the tooth profile is given, it is a simple matter to construct its line of action in regard to any given pitch line, but when the line of action alone is given, it is a much more difficult task to construct the tooth profile. If, in addition to the line of action, we definitely know certain points on it that represent known angular or linear movements of the profile, this problem is greatly simplified.

In order to clarify the foregoing, we will consider several definite problems. With the exception of a few mathematical curves, such problems must be solved geometrically or by actual layouts. In practice, these are made to a sufficiently enlarged scale to obtain the desired degree of accuracy. The following examples indicate the methods employed in making such layouts:

PROBLEM 1.—*Given an arbitrary gear-tooth profile, to construct its line of action.*

The arbitrary tooth profile selected is a straight line, as shown in Fig. 2. The pitch line is established at the middle of the tooth height. This gear has 36 teeth, giving an angular distance of 10 deg. between the successive teeth.

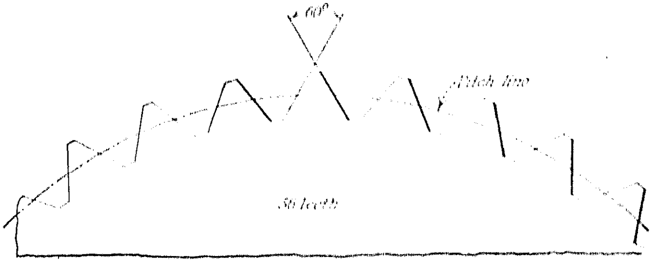

FIG. 2. Straight-line gear-tooth profile.

To construct the line of action, the tooth profile is rotated about its axis into several successive positions. At each position, a line through the pitch point and perpendicular to the tooth profile is drawn. The intersection of this line with its respective tooth profile is a point on the line of action. This point is where contact is made with any mating conjugate profile with the given profile in this position. After determining a series of such points, the line of action can be drawn through them. It is usually a good plan when laying out the successive positions of the tooth profile to make the intervals equal, as this greatly assists later in the construction of mating conjugate profiles.

The construction of the line of action for the selected profile is shown in Fig. 3. On the left, at *A*, are shown the construction lines only, that is, the successive positions of the tooth profile and the perpendiculars through the pitch point to each position of the tooth profile. The intersections, or contact points, are marked with heavy dots. Each position of the tooth profile is numbered for identification later. In this example, the intervals between the successive positions of the tooth profile are equal to 5 deg., or one-half the tooth spacing. In actual practice, the scale of the drawing would be greatly enlarged, and the intervals between the successive positions of the tooth profile would be much smaller.

At the right, in Fig. 3, at *B*, the line of action is shown in a heavy line. A study of this illustration should make clear the method of constructing the line of action from a given gear-tooth profile. When the profile is not a straight line, the tangents and normals to the curved profile must first be determined; otherwise, the process is identical. Usually, the arbitrary profiles are straight lines, arcs of circles, or a combination of the two.

PROBLEM 2.—*Given an arbitrary gear-tooth profile, to construct the profile of its basic rack.*

The arbitrary tooth profile will be the same as before, illustrated in Fig. 2. To construct the profile of the basic rack, a series of contact points must first be determined, as was done to construct the line of action. The pitch line on the rack will be a straight line. A straight line representing this pitch line is drawn, and on it are laid off intervals corresponding in length to the length of the arcs on the pitch circle of the gear between the successive angular positions of the gear-tooth profile. In this example, the lengths are equal to an arc of 5 deg. on the pitch circle of the original gear. The construction is shown in Fig. 4. At *A* is shown the same layout as at *A* in Fig. 3. The intervals along the pitch line of the rack at *B* are numbered in the reverse order to the successive positions of the profile at *A* and represent the successive positions of the pitch point as the original gear is rotated to the corresponding positions. By drawing a series of parallelograms, successive points on the basic-rack profile are has been established for any profile, a definite line of action exists, along which contact with all other conjugate profiles is made. There is a definite relation between a gear-tooth profile and its line of action, so that if either one is given, the other is fixed. When the tooth profile is given, it is a simple matter to construct its line of action in regard to any given pitch line, but when the line of action alone is given, it is a much more difficult task to construct the tooth profile. If, in addition to the line of action, we definitely know certain points on it that represent known angular or linear movements of the profile, this problem is greatly simplified.

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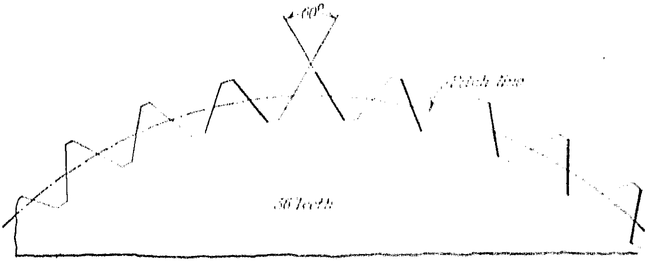

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